NATURE OF FAILURE AND FILTRATION PROPERTIES FOR A POROUS GAS-IMPREGNATED MATERIAL FOLLOWING CAMOUFLET EXPLOSION

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In the case of carrying out a camouflet explosion in a porous gas-impregnated material the nature of change in filtration properties depends markedly on the initial material porosity. With the aim of studying the dependence of the change in permeability caused by a camouflet explosion on the initial material porosity, laboratory experiments have been carried out with artificially prepared models of a porous collector having different porosity values. Results of experiments point to the presence of a qualitative difference in permeability factor after an explosion in materials with porosities $m_0 > 15$ and $< 15\%$. A circulation model is suggested explaining this effect by the change in structure of the pore space. Specific mechanisms leading to occurrence of this effect may be varied: crushing of grains, failure of intergranular cement, etc.

1. Experimental Procedure. Laboratory tests with camouflet explosions of a spherical charge of PETP weighing from 0.5 to 2 g were carried out in materials with an initial porosity $m_0 = 25$, 28, and 10%, whose pores contained air. The laboratory model was a metal cylinder 300 mm in diameter and 350 mm high, along whose axis a channel was formed for placing the charge in the center of the model. The porous material filling the cylinder was a mixture of dressed sand, lime flour, sodium silicofluoride, and water glass. This mixture was pressed at 6 and 14 MPa and held in a furnace for 55 h at about 100°C. Thus, it was possible to obtain a material with a porosity of 25 and 18% and a permeability of 300 and 100 mD respectively. In order to obtain a material with a porosity of about 10%, use was made of a mixture of coarse sand, Portland cement, and water, which after vibration packing was held for 8 h in a steam chamber at 90°C. The permeability of this material does not exceed 10mD.

The main physicomechanical properties of materials with porosity 25 and 18% were crushing strength $\sigma^* = 20$ and 28 MPa, and longitudinal elastic wave velocity $c_\ell = 3000$ and 3500 m/sec respectively; for material with a porosity of 10% they were $\sigma^* \simeq 35$-40 MPa, $c_\ell = 4000$ m/sec.

In the tests a study was made of the nature and state of the material around the explosion cavity and the change in filtration characteristics of the material.

The nature of failure and the condition of the material were studied visually and by direct measurement of material density from the explosion cavity towards the periphery, and the change in filtration properties was determined by comparing the pre- and postexplosion filtration characteristics of the material.

The procedure for visual study of the nature of material failure was thus. After an explosion and recording filtration characteristics, the material was released from the metal shell and sawn in the transverse section in the plane of charge location. The size of the cavity, cracks, and overall state of the material were determined around the cavity, and the condition of the pipe for measurement of filtration properties was determined.

In order to measure material density from the explosion cavity towards the periphery a densitometric method was used for recording scattered γ-radiation in a modification of narrow-beam γ-quanta radioscopy. Measurement of density distribution in the material was carried out in disks up to 5 cm thick sawn from the model after an explosion in the plane of charge application. Measurement was carried out over the radius from the center, and the pitch of measurements was varied from 1 cm at the periphery to 0.5 cm in the central part of the disk.

In order to study the filtration parameters of the material during model preparation at different distances from the charge, eight to nine tubes 3 mm in diameter were installed. The

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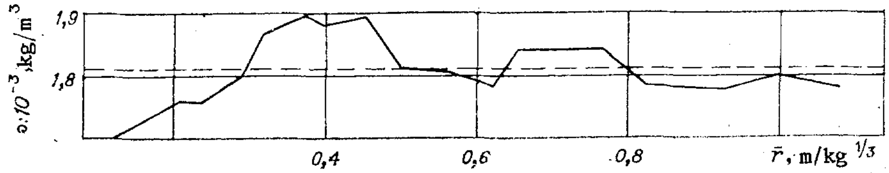

Fig. 1

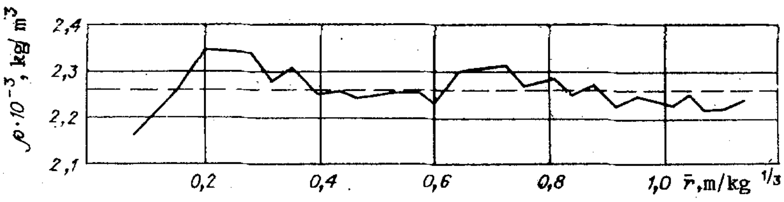

Fig. 2

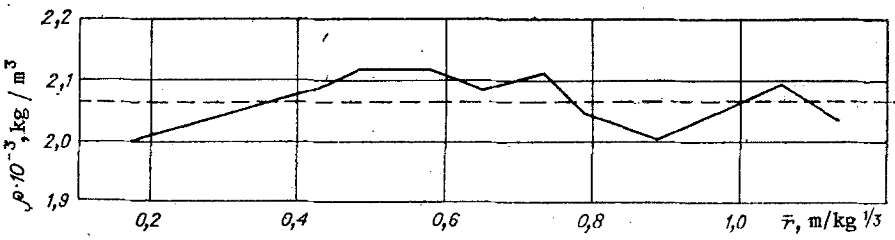

Fig. 3

ends of the tubes were perforated over a length of 18-20 mm, and their opposite ends emerging from the model were joined with a measurement circuit. In test the steady input Q of air was determined and the pressure drop Δp corresponding to it between pairs of tubes before and after the explosion. As a filtration characteristic use was made of the relationship $\Gamma = Q/\Delta p^2$, the change in filtration properties of the material as a result of the explosion was estimated by the ratio Γ/Γ_0 (index 0 relates to preexplosion tests). A similar procedure was given in [1].

2. Results of the Experiments. In all of the cases around the cavity, which in shape is close to spherical, a zone separates with variable color which may be determined as pressed, but weakly-bonded material in the form of an aggregate of material grains. Cracks, colored by explosive gases, follow along boundaries of the cavity and they intersect the zone of pressed material. Their general number is three to five, opening less than 0.5 mm. Filtration tubes did not suffer damage.

Results of studying material density from the cavity towards the periphery, averaged over the whole radius for a given material, are shown in Figs. 1-3 for m_0 = 25, 18, and 10% respectively (broken lines are average density, equal to the mean arithmetic value of the total number of measurements and taken for the initial material density). It can be seen that successively from the cavity for materials with different original porosity there are separate zones of loosening and densification. The extent of the densification zone increases as the original material porosity decreases. The density of the material in the loosening zone decreases by 15%, and in the densification zone it increases on average by 5-10%. Material deformation revealed is shown in Table 1.

Shown in Figs. 4-6 are functions of the change in gas impregnated material permeability with a camouflet explosion for materials with initial porosity 25, 18, and 10% respectively.

For materials with porosity m_0 = 25% the postexplosion filtration properties deteriorate to $\bar{r}$ = 2 m/kg$^{1/3}$, and the minimum permeability is observed close to the cavity (by a factor of ~10^2). In material with porosity m_0 = 18% in the range of distances $\bar{r}$ = 0.24-1.4 m/kg$^{1/3}$ (the outer boundary) the postexplosion filtration properties also deteriorate with a maximum at $\bar{r}$ = 0.24 m/kg$^{1/3}$ by a factor of about four, but closer than $\bar{r}$ = 0.24 the change in filtration properties was not studied. A reduction in porosity to m_0 = 10% leads to a radical change in the mechanisms for variation of postexplosion filtration properties. The nonuniform curve in Fig. 6 contains three regions successively from the cavity with different nature of change in filtration properties and it lies above the original preexplosion values. The first region, whose radius $\bar{r} \approx$ 0.3 m/kg$^{1/3}$, is characterized by a reduction in permeability from the center to the boundary, the second region has a minimum for the increase

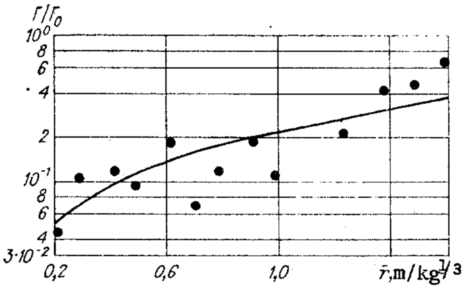

Fig. 4

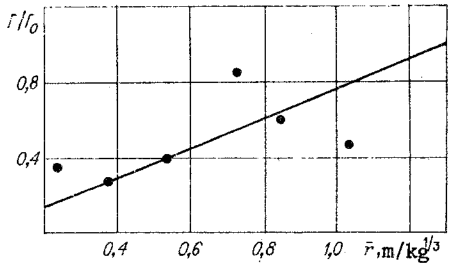

Fig. 5

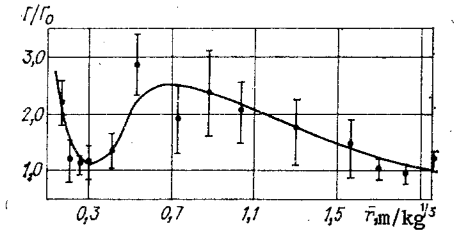

Fig. 6

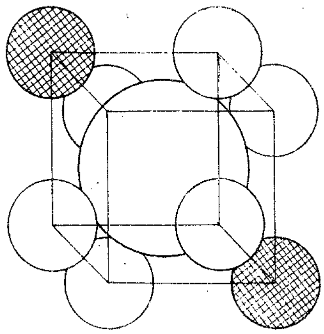

Fig. 7

TABLE 1

Initial porosity m_0, %	Zone size, m/kg$^{1/3}$				
	cavity radius $\bar{r}_p$	intense failure radius (change in color) $\bar{r}_{i.f}$	radius of new cracks $\bar{r}_c$	loosening radius $\bar{r}_\ell$	Densification range $\bar{r}_d$
25	0.12	0.26	0.4	0.3	0.3—0.5
18	0.1	0.27	0.6	0.2	0.2—0.35
10	0.08	0.29	0.8	0.3	0.3—0.9

in permeability, and the third has an outer boundary at $\bar{r}$ = 2.0 m/kg$^{1/3}$ and it embraces a considerable volume of material with improved filtration properties.

From the data presented attention is drawn to the following facts. It appears paradoxical that in spite of loosening close to the cavity for material with m_0 = 25 and 18%, a deterioration in permeability is observed, and in the loosest part it is at a maximum. Beyond the densification zone in all cases a change is noted in filtration properties, although changes in the material are not observed by other methods. Finally, a further reduction in porosity leads to a change in regularities for the variation of material filtration properties.

3. Percolation Model for the Change in Permeability of a Granular Material. In order to explain by experiment the observed effect we consider in detail the structure of the pore space of a collector which is an assembly of grains joined by a cementing substance. One part of the intergranular space is occupied by cement, and the other is the pore space of the collector. A percolation model for permeability of granular materials was suggested in [2]. From the point of view of percolation theory the intergranular space is assemblies joined by narrow channels (bonds). Some of the assemblies are plugged with cement substance, and the rest have a capacity to pass fluid. Percolation theory establishes that with a certain fraction x of conducting assemblies in an infinite material continuous chains of conducting assemblies occur (infinite cluster) along which it is possible to accomplish fluid filtration. With a lower fraction the infinite cluster does not form and filtration is impossible. This critical value is called the flow threshold P_c, and it depends on topology of the problem, i.e., its size and arrangement structure for assemblies in space. In the region $x > P_c$ permeability of a granular medium is described by the equation [2]

$$K = 2\cdot 10^{-3}\frac{m_0^3 d^2}{(1-m_0)^2}(x-P_c)^t\,\eta\,(x-P_c). \tag{3.1}$$

Here $m_0 = 1 - V_3/V$; V_3 is the volume occupied by grains; V is the overall volume of the medium (we call m_0 structural porosity); x is a fraction of conducting intergranular assemblies of their total number (with random distribution in space for conducting and nonconducting assemblies the value of x equals the probability of any assembly being conducting); t is an index depending only on the size the of the problem (for a three-dimensional space $t = 1.7 \pm 0.2$); $\eta(x - P_c)$ is a Heaviside unit function; d is grain diameter.

With passage of a shock wave (SW) caused by a camouflet explosion there may be failure of a very weak area of intergranular contacts; either plastic flow of the cementing substance or failure of intergranular contacts completely free of cement. The specific mechanism in one case or another depends on the structure and composition of rock grains and the intergranular cementing substance. Failure of intergranular contacts may lead to filling of the adjacent pore space by failure products. Thus, there is rebuilding of the structure of the pore space and a change in filtration properties of the medium. In future for definiteness the reason for filling of intergranular assemblies of the pore space will be assumed to be failure of grains at contacts between which cement is absent.

In carrying out a camouflet explosion the material in the crushing zone breaks down into blocks whose subsequent movement as a result of the effect of dilation leads to occurrence of interblock porosity. The resulting filtration properties of the material are governed by the sum of permeability for blocks and interblock space (dilation permeability). For low-porosity rocks the proportion of intergranular contacts entirely free of cement is negligibly small. Correspondingly, rebuilding of the structure of the pore space gives a negligibly small contribution to the change in block permeability. The main mechanism for improving filtration properties of low-porosity materials is dilation dispersion [3], which explains the improvement in permeability coefficient in the whole of the region experiencing the effect of the explosion.

In a material with high initial porosity there are many intergranular contacts completely free of cementing substance. In this case the permeability coefficient for blocks may markedly exceed dilation permeability, which rapidly decreases with an increase in distance, and it makes an insignificant contribution to the change in filtration properties of the material over almost the whole region affected by the explosion, with the exception of a narrow zone directly adjacent to the camouflet cavity in which there is severe crushing of the material and displacement of it. The size of this zone is easily found from the inequality $K_d(r) < K_0$, where K_d dilation permeability determined in [3], and K_0 is initial material permeability. Beyond this region of strong crushing of collector material with passage of an SW caused by an explosion, grains having cement-free contacts fail, and the failure products of a grain fill the surrounding pore space. Thus, there is a reduction in conducting porosity due to rebuilding of the pore space structure, which in turn causes a deterioration in permeability.

First we consider the simplest case of regular packing of spherical grains of a single radius in a simple cubic lattice. Structural porosity is partly occupied by intergranular cement and it is partly free. The free part of structural porosity is the pore space of rock-collector. The amount of true porosity is designated in terms of m. Then $x = m/m_0$ is the fraction of structural porosity free from cementing substance.

On the other hand, x is the probability that an assembly is conducting. We consider this packing from the point of view of percolation theory. Each grain embraces eight intergranular assemblies whose centers lie at the corners of a cube (Fig. 7). It can be seen that intergranular assemblies form a normal system with a simple cubic lattice. These assemblies are either plugged with cement and impenetrable, or free from cement. A collection of assemblies free from cement is a conducting cluster. The proportion of conducting assemblies of their total number is $x = m/m_0$. Depending on the value of x the material may be conducting or not. The critical value for the problem of assemblies in a cubic lattice $x_c = 0.31$, and the value of structural porosity $m_c = 0.48$ [2].

Failure of a grain with probability α will occur if in one face of the cubic lattice shown in Fig. 7 all four assemblies do not contain cement. Fragments of a failed grain fill all of the free assemblies, and this changes the structure of a conducting cluster. We determine probability P such that a grain has a free face. The probability of failure of a

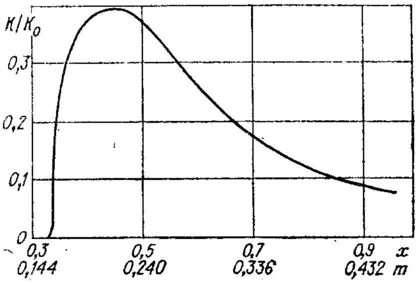

Fig. 8

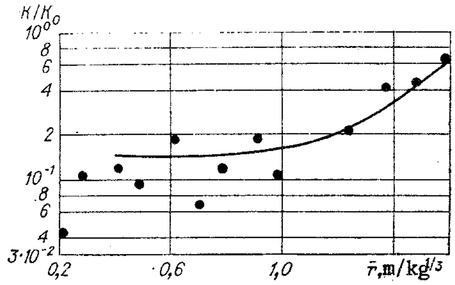

Fig. 9

grain is governed by the total probability of five independent occurrences: when all eight assemblies, seven, six, five, and four appear empty. We designate the probability of these occurrences in terms of P_8, P_7, P_6, P_5, P_4. Probability $P_8 = x^8$, since the coincidence of eight independent occurrences for which the probability of each is x. Correspondingly $P_7 = x^7(1 - x)$. Since an assembly filled with cement will be any of the eight assemblies embraced by a grain, then in calculating P it is necessary to add $8P_7$ to P_8. Probability $P_6 = x^6(1 - x)^2$. Of all of the possible combinations of eight assemblies, for two of them it is necessary to exclude four of these when two cemented assemblies are located at opposite ends of a cube diagonal, since in this case there is not one face in which all four assemblies would be empty. One such possible combination is presented in Fig. 7 where the assemblies occupied by cement are hatched. It can be seen that with this location of cemented assemblies not one face of the lattice is completely free from cementing substance. Then the contribution to the probability of grain failure of a combination of six empty and two cemented assemblies will be $24P_6$. Similarly the contribution of a combination of five empty and three filled assemblies gives $24P_5$, where $P_5 = (1 - x)^3x^5$, and from a combination of four empty and plugged assemblies we have $6P_4$ where $P_4 = x^4(1 - x)^4$. Thus, probability $P = P_8 + 8P_7 + 24P_6 + 24P_5 + 6P_4$.

Since with failure of a grain there is plugging of three assemblies, then it is necessary to determine the average proportion of newly plugged assemblies occurring in a single grain P_1. By knowing P it is easy to understand that

$$P_1 = \frac{1}{8}(8P_8 + 7\cdot 8P_7 + 6\cdot 24P_6 + 5\cdot 24P_5 + 4\cdot 6P_4). \tag{3.2}$$

The multiple appearing ahead of the terms in brackets is determined by the number of empty assemblies in corresponding cases. Since in the fraction of one lattice cell there is one eighth of each assembly, then all of the bracket is multiplied by 1/8. By knowing P_1, it is possible to determine the fraction of assemblies free from cement after failure $x' = x - P_1\alpha$ (α is the probability of failure of a grain having a face free from cement). Now it is easy to find the change K/K_0 after an explosion by using an expression for the converted fraction of conducting assemblies $x' = x - \alpha P_1$. As follows from Eq. (3.1),

$$\frac{K}{K_0} = \left(\frac{x' - P_c}{x - P_c}\right) = \left(1 - \frac{\alpha P_1}{x - P_c}\right)^t, \tag{3.3}$$

where P_1 is determined by Eq. (3.2). The dependence of relative change in permeability coefficient on the value of x is given in Fig. 8 for $\alpha = 0.5$. Since m_0 of the material is clearly connected with the probability of conducting for an assembly $x = m/m_0$, then relationship (3.3) is the dependence of change in filtration properties for the material on initial porosity. With the aim of illustrating this dependence corresponding values of porosity are applied to the horizontal axis.

It can be seen from Fig. 8 that at a distance from the threshold filtration characteristics deteriorate uniformly. Physically this is connected with the fact that with an increase in initial porosity there is an increase in the number of intergranular contacts free from cementing substance, whose failure causes rebuilding of the pore space structure. The greater the "bare" contacts, the stronger is the effect of a change in pore space and the more permeability is reduced. A sharp deterioration in permeability close to the threshold is explained by the fact that in this region in a permeable material there are very few conducting chains comprising an infinite cluster, but the number of contacts free from cementing substance is sufficiently large that in failing there is a reduction in the proportion of conducting assemblies to a value below the threshold P_c.

In the case being considered packing in the form of a simple cubic lattice for a material only becomes permeable with $m > m_* = 0.15$ (m_* is critical porosity corresponding to the threshold for the material [2]).

Apart from the simplest case of periodic packing of spheres in a cubic lattice, other packing methods are also possible: pacing in the form of bcc and fcc lattices. Corresponding threshold values of porosity are for the bcc lattice $m_* = 8\%$, and for the fcc lattice $m_* = 5\%$. For the case of regular packing of grains in the form of bcc or fcc lattices relationship (3.3) remains in force, in which there is only a change in numerical values of P_c and P_1. All of the qualitative aspects of the dependence for the change in permeability on amount of initial porosity are entirely retained.

An actual granular material is a mixture of grains of different sizes. At different size levels in an actual material different types of grain packing may be realized in small volumes. However, independent of material packing with $m_0 < 0.05$ they will behave during an explosion as a monolith since conducting chains of intergranular assemblies are absent from them and the change in pore space structure does not cause a change in block permeability. In a material with $m_0 > 0.15$ independent of actual grain packing there is always a considerable number of conducting intergranular assemblies which form a branched infinite conducting cluster. During the action of an explosion in such a material there is considerable rebuilding of the pore space (since with an abundance of conducting assemblies in the material there are many intergranular contacts completely free from cementing substance). This leads to a marked deterioration in filtration properties. On the other hand, the initial permeability of highly porous materials ($m_0 > 0.15$) considerably exceeds the dilation permeability everywhere, with the exception of a small region directly adjacent to the camouflet cavity. Therefore, a change in the initial permeability of the material due to rebuilding of the pore space has a dominant effect and it governs the resulting deterioration in material permeability.

Materials with initial porosities from 0.05-0.15 form a class with transitional properties. Behavior of the permeability coefficient after a camouflet explosion in these materials is determined both by the mechanism of dilation dispersion of blocks and a rebuilding mechanism for the pore space within blocks.

It is interesting to note that within the range from 5 to 15% there is a typical value $m_* \sim 8\%$ corresponding body-centered packing. With $m_0 > 8\%$ stresses in the SW may lead to rebuilding of the pore space structure in the material with this type of packing. Therefore, in the range from 8 to 15% the effect of an SW on granular materials lead to a more marked change in material permeability compared with the range from 5 to 8%.

These values of typical porosities 5, 8, and 15% are approximate since values of the flow thresholds are known with an accuracy of the order of 2%. Correspondingly the inaccuracy of determining m_* reaches one per cent, the actual value of m_* is found within the limits ±0.01.

4. Radial Dependence for the Change in Permeability Coefficient for a Granular Material After an Explosion. The physical reason for the failure of grains is the SW caused by the explosion. In a real material there are grains of different size. We shall assume that grains have a shape close to spherical, and grain diameter distribution is described by a distribution function $f(a)$ which is normalized to unity. According to the Hall–Petch rule [4] the strength of a grain is inversely proportional to the square root of grain size. Since with propagation of an SW from the center of an explosion its intensity decreases, then with an increase in distance the minimum size of failed grains increases. Thus, the proportion of grains which may be failed by stresses in the SW front decreases with an increase in distance from the center of the explosion. Correspondingly to a lesser extent there will be rebuilding of the pore space, and consequently less change in permeability coefficient. We consider function $f(a)$ in the form

$$f(a) = \frac{a}{a_c^2} \exp(-a/a_c).$$

For the function $f(a)$ selected the proportion of grains with a size exceeding is

$$P_2 = (a/a_c + 1) \exp(-a/a_c).$$

In view of the Hall–Petch rule only those grains are subject to failure by stresses whose dimensions exceed $a \geq A/\sigma^2$, where A is a constant. The dependence of stress σ at the SW front on distance was determined by experiment for a broad class of rocks and rock-collectors [1]. As experiments showed, stresses at the SW front attenuate with distance by a power rule $\sigma = B(r_0/r)^\beta$, where β in the region of interest to us may be made equal to two. Now the dependence of minimum failed grain size on distance from the explosion center is written in the form

$$a(r) = D\left(\frac{r}{r_0}\right)^{2\beta},$$

where $D = A/B^2$. The proportion of grains with a size exceeding $a(r)$ is

$$P_2 = \left(\frac{Dr^{2\beta}}{a_c r_0^{2\beta}} + 1\right)\exp\left(-\frac{Dr^{2\beta}}{a_c r_0^{2\beta}}\right). \tag{4.1}$$

Since of all grains located at a distance r from the center of the explosion, only P_2 of the total number may fail, then in Eq. (3.3) for the change in permeability $\alpha = P_2$

$$\frac{K}{K_0} = \left[1 - \frac{P_1\left(\frac{Dr^{2\beta}}{a_c r_0^{2\beta}} + 1\right)\exp\left(-\frac{Dr^{2\beta}}{a_c r_0^{2\beta}}\right)}{x - P_c}\right]^t. \tag{4.2}$$

Relationship (4.2) determines the change in permeability of a porous gas-impregnated material as a function of distance from the center of the explosion. A curve for relationship (4.2) is presented in Fig. 9 for $P_1 = 0.21$, $x = 0.5$, $P_c = 0.2$, $D/a_c = 10^{-4}$, $\beta = 2$. Comparison of the curve given in Fig. 9 with an experimental curve (see Fig. 4) for $m_0 = 25\%$ indicates good conformity of theory and experiment.

Results of an experimental study of the dependence of permeability coefficient on amount of initial material porosity after carrying out a camouflet explosion indicates that with an increase in initial porosity there is a change in the nature of behavior of permeability from an improvement in filtration characteristics to deterioration. Besides this, close to the cavity of a camouflet explosion there is always some material dispersion characterized by a reduction in the average density close to the cavity.

A theoretical model for the change in filtration properties after an explosion based on ideas of percolation theory explains the dependence observed in the experiment for permeability after an explosion on initial porosity by changing the structure of the pore space under the action of an explosion. With low porosity ($m_0 < 0.05$) the effect of a change in pore space structure is not indicated in the permeability of blocks and the change in permeability is governed by dilation dispersion of the material.

With an increase in initial porosity the change in pore space structure starts to make a more marked contribution to the overall change in permeability.

For material with porosities in the range from 0.05 to 0.15 the mechanisms of dilation dispersion and pore space rebuilding contribute to the change in permeability. In highly porous materials ($m_0 < 0.15$) the relative deterioration in permeability due to rebuilding of the pore space structure predominates over dilation improvement of filtration properties. This explains the resulting deterioration of permeability for highly porous materials after an explosion.

In the USA powerful explosions have been carried out in gas-bearing seams by the Gasbuggy and Rio-Blanca projects. Explosions by the Rio-Blanca project were carried in a gas-bearing seam with $m_0 = 26\%$, and by the Gasbuggy project in a gas-bearing seam with $m_0 = 11.8\%$. As a result of a Rio-Blanca explosion the permeability of the material deteriorated [6], in spite of loosening close to the cavity [7]. With a Gasbuggy explosion the permeability increased up to distances of $\bar{r} = 0.3$ m/kg$^{1/3}$ from the charge [5]; American researchers suggested that beyond these distances seam permeability was unchanged compared with the background value.

The data provided indicate an identical character for failure and change in filtration properties of a material with identical initial porosity in full-scale and laboratory tests, which confirms the objectivity of the conclusions obtained in this work.

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A CLASS OF SINGULARLY PERTURBED BOUNDARY VALUE PROBLEMS IN A TWO-DIMENSIONAL ELASTICITY THEORY

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UDC 539.3

In studying boundary value problems for an elastic medium reinforced by a family of very stiff fibers, authors are frequently concerned with the limiting case model of an elastic medium with inextensible fibers, i.e., deformation along a specified direction is equal to zero [1]. With the exception of [2], little attention has been devoted to ascertaining the correctness of this limiting model; in [2], under rather stringent assumptions concerning the boundary of the domain, consideration was given to a boundary value problem with a specified stress vector on the boundary for a medium inextensible in a given direction and with direct reinforcement.

Here we prove a series of theorems concerned with the convergence of singularly perturbed problems of a given class to limiting solutions in corresponding Hilbert spaces; we also show that the limiting system of equations cannot coincide with the system resulting from the assumption of inextensibility. A concrete example of a similar situation appears in [3].

1. In a curvilinear orthogonal coordinate system in the plane, (α_1, α_2), we take the generalized Hooke's Law for an orthotropic material in the form [4]

$$\sigma_{11} = \varepsilon^{-2}e_{11} + b_{12}e_{22},\ \sigma_{22} = b_{12}e_{11} + b_{22}e_{22},\ \sigma_{12} = 2e_{12}, \tag{1.1}$$

where the dimensionless stresses are taken with reference to the shear modulus G; the axes of orthotropicity coincide with the (α_1, α_2) axes; $\varepsilon^{-2} = b_{11}G^{-1} \gg 1$; $\varepsilon \ll 1$; $b_{22} - \varepsilon^2 b_{12}^2 > 0$; $b_{22} > 0$. Deformations may be represented in terms of displacements $u = (u_1, u_2)$ in the following way:

$$e_{11} = \frac{1}{h_1}\frac{\partial u_1}{\partial \alpha_1} + \frac{1}{h_1 h_2}\frac{\partial h_1}{\partial \alpha_2}u_2,$$

$$e_{22} = \frac{1}{h_2}\frac{\partial u_2}{\partial \alpha_2} + \frac{1}{h_1 h_2}\frac{\partial h_2}{\partial \alpha_1}u_1,$$

$$2e_{12} = \frac{h_2}{h_1}\frac{\partial}{\partial \alpha_1}\left(\frac{u_2}{h_2}\right) + \frac{h_1}{h_2}\frac{\partial}{\partial \alpha_2}\left(\frac{u_1}{h_1}\right),$$

h_1 and h_2 are the Lamé parameters (h_1, $h_2 \geq$ const > 0, $h_{k.\alpha_j}$ (k, j = 1.2)), measurable and bounded in a compact simply connected domain Q with a piecewise-smooth boundary γ. We introduce the Hilbert space V of functions $v = (v_1, v_2)$, $v_k \in L^2(Q)$ (k = 1.2) with the finite norm

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